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Progressive Degenerate Ellipsoidal Phase Plug

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ABSTRACT

This paper will detail the concepts and design of a new phase plug. This device can be utilized to transform a circular, planar wave front to a rectangular, planar wave front. Such functionality can be very useful for line array applications as well as for feeding the input, or throat section, of a rectangular horn from the output of conventional compression drivers. The design of the phase plug allows for the exiting wave front to have either concave or convex curvature if a planar wave front is not desired. One of the novel features of this device is that there are no discontinuities within the phase plug.

1. INTRODUCTION

Since the introduction of commercially viable, large scale line array loudspeaker systems over 20 years ago there have been many different designs of wave shaping devices to yield flat (planar) wave fronts at their exits. The shape of the wave front that is radiated from the exit of a wave guide or phase plug is of paramount importance to the coupling of the sound radiated from a device with that of its neighbors, particularly in the higher frequency regions where the wavelength of the radiated sound waves are small [1].

It is important that, at a given point in space, the radius of curvature is the same for wave fronts radiated by adjacent devices. These wave fronts should also be tangential to each other. This minimizes or eliminates the destructive interference that can occur when the same signal is radiated from two or more devices, each at different spatial locations. Properly designed wave front shaping devices are an absolute necessity for the output of adjacent loudspeaker modules (boxes) to sum together optimally in the higher frequency regions (i.e. at shorter wavelengths) with a minimum of destructive interference from an array of these devices.

2. BACKGROUND

For the design and operation of loudspeaker systems, particularly those intended to be arrayed, it can be advantageous to transform the shape of the wave front that is radiated from a given device to have a different shape or different boundary conditions confining the wave front, or both. Such is the case with devices from Heil [2] and Adamson [3]. Both of these inventions utilize a cone as a central element within a similar, cone-shaped cavity to transform a circular planar wave front radiated by a compression driver into a rectangular planar wave front. A rectangular planar wave front is better suited to drive the input of certain horn designs as well as for use in line array applications.

The inner core of both the Heil and Adamson devices is formed by cutting the cone with two planes, each going through the diameter of the base and the midpoint of the cone along its height. This results in a surface that is cone-shaped at one end (where the apex resides) and wedge-shaped at the opposite end (where the base of the cone formerly existed). An edge now exists along the line that was the diameter of the base of the cone. The demarcation between each of these sections is a parabola where the planes cut the cone. The surface that is formed for the Heil device has a particularly useful property. When the straight-line distance from the apex of the cone to any point on the parabola is added to the straight-line distance from that point on the parabola to the edge (such that the straight line is normal to the edge) a certain length is the result. This length is identical to any other similarly derived path going through any other point on the parabola (Figure 1). Thus the shortest length traveled for any path from the apex of the cone to the edge at the opposite end is the same regardless of the path traveled.

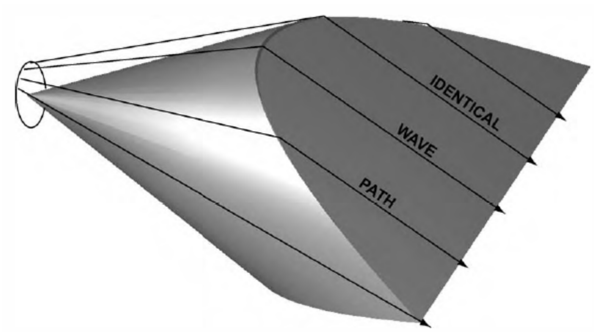

Figure 1: Heil phase plug

The surface of the Adamson device differs slightly in that the original cone is not circular at its base, but is instead elliptical. The cutting planes intersect the semi-major axis, instead of the diameter of a circular base. This allows for a path length closer to the middle of the surface to be increasingly shorter than a path length farther away from the middle of the surface. This allows for a convex wave front to be formed at the exit.

A cavity may be created by placing an outer shell, of the same shape as either of the above described inner cores, around either of the inner cores, respectively. From an acoustical perspective, the equal path lengths through the cavity of the Heil device have the property of preserving the curvature of the wave front presented at the entrance of the cavity (apex of the cone). In other words, if a planar wave front was present at the entry of device a planar wave front is radiated from the exit of the device. It is only the outer perimeter confining the wave front that has changed, in this case from circular, at the entrance, to rectangular at the exit

The same occurs in the case of a cavity formed with the Adamson core. However, the curvature of the wave front at the exit may be altered from that presented at the entrance by altering the aspect ratio for the base of the elliptical cone. This can also have benefit in that a non-planar wave front with defined curvature may result.

One potential disadvantage of both the Heil and Adamson devices is the discontinuity that is formed on the conical surface by the cutting planes. This parabola shaped edge may give rise to diffraction within the cavity formed by the inner core and outer shell. Reducing or eliminating the discontinuity may yield improvements to these very elegant solutions to wave front boundary reshaping.

3. NEW PHASE PLUG CONCEPT WITH EQUAL PATH LENGTHS

3.1. Concept

The new phase plug is constructed from an inner core surrounded by an outer shell. The air space between these two elements creates a conduit through which pressure waves from a transducer can freely propagate.

The shape of the new phase plug is similar to that of both the Heil and Adamson devices, but with a smooth transition between the cone section and the wedge section. The shape of the new phase plug can be thought of as having three different sections. The first section is that of a cone, with an apex at one end (entrance). The last section is wedge-shaped at the opposite end (exit) from the apex. These two sections are joined by a middle section made from arc lengths of different ellipses.

A key feature that eliminates discontinuities in the shape of the phase plug is that the section at each end of the phase plug, both cone and wedge sections, must be tangent to the middle section (formed by ellipses) at the point where the sections join together. This requires that multiple different ellipses are used to join the cone and wedge sections. These different ellipses are calculated based on different angular cross sections of the phase plug with respect to the horizontal plane (Figure 2).

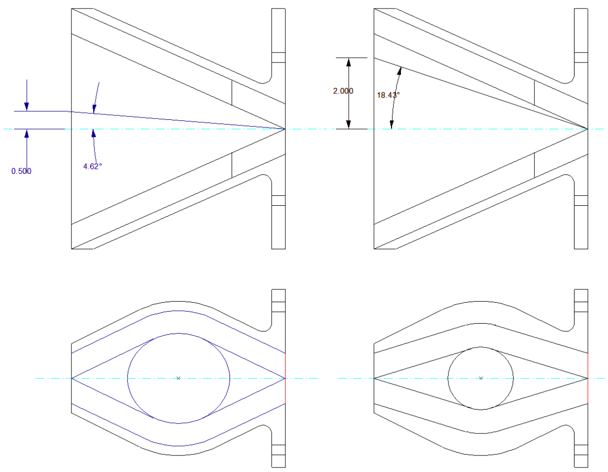

Figure 2: Two different angular cross sections (upper) and the normal projection to each of the angular cross sections (lower)

As the angle of each cross section increases, the ellipse used to join the cone and wedge section can be thought of as progressively degenerating to a line, thus the name of the device. Once the equations were developed to solve for the parameters of each ellipse required for the design of practical phase plugs, it was discovered that these ellipses do not actually degenerate to a line, but to a point.

3.2. Creating the Required Shape

To determine the shape of the phase plug's inner core, some definitions for the geometry must be established. The first set of these can be seen in Figure 3, which is a side view (elevation) for the vertical cross section of a phase plug. Here we can see the following.

d is the entry diameter of the device

H_{exit} is the total height of the exit of the device

H_{core} is the height of the inner core at the exit of the device

φ_{max} is the maximum angle of the top or bottom surface from the center line of the device

r_{max} is the length of the top or bottom straight line portion of the device

L is the overall horizontal length of the device

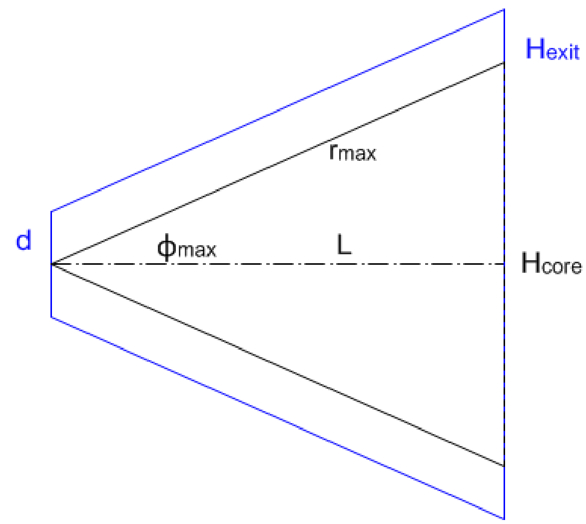

Figure 3: Side view of basic phase plug geometry

From this we can derive the following relationships.

$$\varphi_{\max} = \tan^{-1}\left(\frac{H_{core}/2}{L}\right) \tag{1}$$

$$r_{\max} = \frac{L}{\cos(\varphi_{\max})} \tag{2}$$

The value of r_{max} is important as it represents the longest straight-line distance from the entry to the exit of the device. The path along L, where φ is 0°, is a much shorter distance. It is the distance of this path, and all others paths within the range $0° \leq \varphi < \varphi_{max}$, that must be increased to make them equal to that of r_{max}. This is accomplished by using an ellipse and lines tangent to an ellipse, to form the increased path length. A different ellipse will be required for each incremental value of φ.

When φ equals φ_{max} an ellipse is no longer needed as the path is the straight line of r_{max}.

We will now develop the relationships to create a path length, F, which lies in a plane defined by a cross section at an angle φ. The length of F must be equal to r_{max} for a planar wave front at the exit of the phase plug. F will be comprised of a portion of an ellipse and line segments tangent to the ellipse.

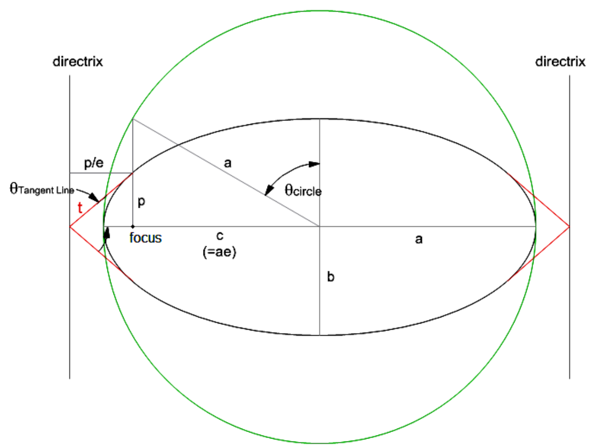

Figure 4: Geometry in a cross section of angle φ

The geometry of a cross section at φ = 0° is shown in Figure 4. The tangent lines, t, create the conical apex at one end and the wedge shape at the other end of the phase plug when viewed from the top. We set the distance between the two directrices of the ellipse equal to the length of our device in the plane of interest (i.e. the cross section at φ = 0° for this figure shown). We define this length as L_φ. This allows us to write the following relationship.

$$L_\varphi = 2\left(c + p/e\right) \tag{3}$$

With the well-known relationships for the semi-latus rectum, p, and the eccentricity, e, of an ellipse we can solve for the semi-minor axis, b, of the required ellipse as a function of the semi-major axis, a, and the length of the device in the cross section, L_φ.

$$p = b^2/a \tag{4}$$

$$e = c/a \tag{5}$$

$$b = \sqrt{a^2 - 4a^2/L_\varphi^{\,2}} \tag{6}$$

With the overall length of the phase plug fixed, equation (6) completely defines an entire series of different ellipses based on the value of the semi-major axis of the ellipse.

Once b is calculated for a particular value of a, the value of all the other parameters of an ellipse may be calculated. We can use this to calculate c, e, and p. These values determine the location of the semi-latus rectum of the ellipse (p in Figure 4), whose endpoints are where the tangent line, t, intersects the ellipse. Since we can calculate the values for p and e we can calculate the angle of the tangent line, $\theta_{TangentLine}$, with respect to the semi-major axis, a. We can also calculate the length of the tangent line, t, from the directrix to the point of intersection on the ellipse. Each of these equations is given below.

$$\theta_{TangentLine} = \tan^{-1}(e) \tag{7}$$

$$t = \sqrt{p^2 - \left(p/e\right)^2} \tag{8}$$

3.3. The Arc Length of an Elliptical Segment

There is no known closed form solution for calculating the perimeter of an ellipse. This could make calculating the length of only a segment of an ellipse troublesome. However, Cantrell developed a very close approximation for the perimeter of an ellipse [4]. This can be used to find the arc length for a section of an ellipse. This approximation is valid for an arc length defined by a point on the ellipse and the nearest intersection of the semi-minor axis, b. Again referencing Figure 4, the arc length of the ellipse between the tangent lines, t, on each side of the semi-major axis, b, is the length needed.

There is a typographical error in equation (8).
The two terms should be added, not subtracted.
The correct equation is:
t = (p^2 + (p/e)^2)^0.5

There is a typographical error in equation (9). The latter term should be multiplied, not added. The correct equation is:
$S = a * (\sin\theta + (\theta - \sin\theta)) * (b/a)\hat{}(2-0.216 * \theta\hat{}2)$

$$S = a * \left(\sin\theta_{circle} + \left(\theta_{circle} - \sin\theta_{circle}\right)\right) + \left(b/a\right)^{\left(2-0.216*\theta_{circle}{}^2\right)} \qquad (9)$$

We will define θ_{circle} as the angle between the semi-major axis, b, and a line connecting the center of the ellipse with the point on the circumscribed circle at which the projection of the semi-latus rectum, p, intersects the circle. Cantrell's approximation for the arc length, S, from the intersection of the tangent line, t, to the semi-major axis, b, is given by equation (9), with θ_{circle} given in radians.

The path length from the beginning of tangent line, t (at the intersection of the directrix), to the end of the other tangent line, t (at the intersection of the other directrix), is given by

$$F = 2(t + S) \qquad (10)$$

As previously stated, by setting the following condition we will have the required equal path lengths need for the phase plug.

$$F = r_{\max} \qquad (11)$$

It would be desirable to find a solution for the semi-major axis, a, as a function of r_{max}. However, since S is dependent on a, b, and θ_{circle} (which is also dependent on a and b) it would be very cumbersome, if not impossible, to derive an analytic solution for a as a function of r_{max}. Therefore, each ellipse which is to be used to join the cone-shaped section at one end and the wedge-shaped section at the other end of the phase plug must be calculated individually based on the value of each angular cross section, φ, as it varies from 0° to φ_{max}.

Once each of these ellipses has been calculated, the inner core of the phase plug can be constructed by smoothly joining each of these cross sections to the adjacent cross sections. The resulting shape might resemble that shown in Figure 5. It can be beneficial to use small angular increments between cross sections so that the interpolation between them does not introduce distortions from what the actual calculated shape should be.

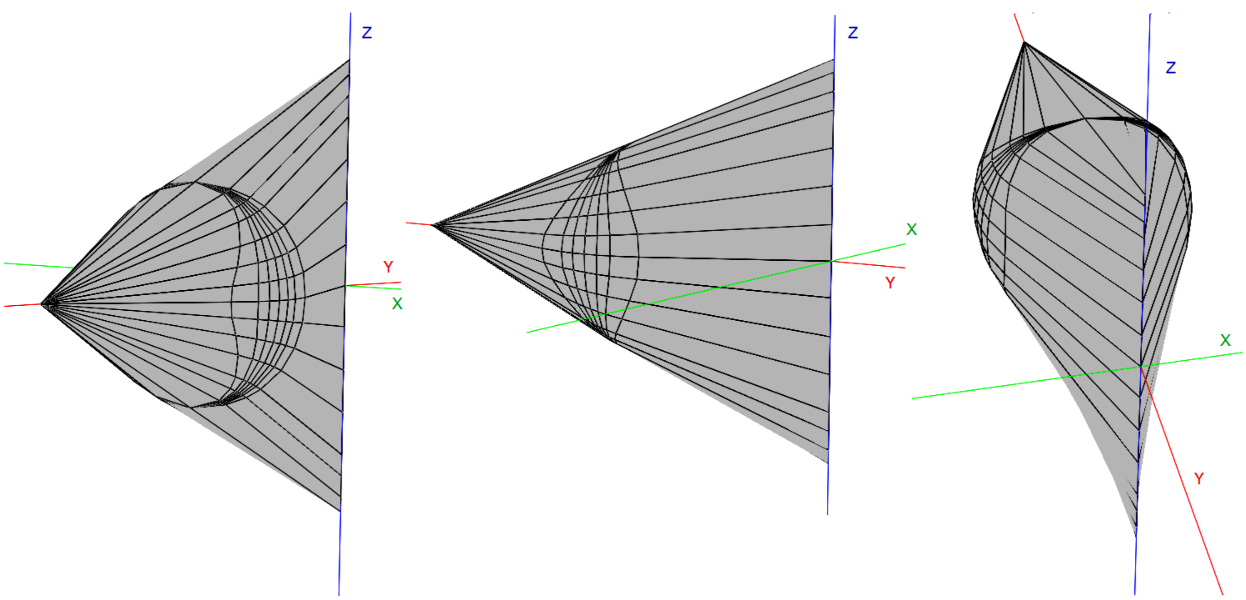

Figure 5: Shape of an inner core for a phase plug

4. CURVED WAVE FRONT AND THE REQUIRED PATH LENGTHS

If a difference in the curvature of the wave front at the entry and exit of the phase plug is desired, this change can easily be incorporated into the design of a device. It may often be convenient to specify the angular curvature of the wave front instead of the radius of curvature. Once the height of the device has been chosen, this angular curvature of the wave front can be used to calculate the radius of curvature. This, in turn, can be used to calculate the change in path length needed to realize the desired wave front curvature. Each ellipse can then be calculated to yield this modified path length.

For a convex wave front at the exit of the phase plug the path lengths along angles less than φ_{max} must be shorter than the path length of r_{max}. Conversely, for a concave wave front the path lengths along angles less than φ_{max} must be longer than the path length of r_{max}.

We define the desired angular curvature of the wave front at the exit of the device as α. Based on the height of the inner core, H_{core}, for the phase plug and this angular curvature of the wave front, the radius of curvature, R_{WF}, for the wave front is determined from equation (12).

$$R_{WF} = \frac{H_{core}/2}{\sin(\alpha/2)} \tag{12}$$

At each angular increment $0° < \varphi < \varphi_{max}$ the height of the inner core at the exit of the device should be calculated. We'll call this height j_i. Alternatively, incremental heights, j_i, between 0 and H_{core} may be specified and the incremental angle, φ, calculated.

Whichever way is chosen, Figure 6 and equations (13) through (16) are used to calculate the required change to the path length at each angular increment, that would otherwise be equal to r_{max}, in order to yield the desired wave front curvature.

$$m = \sqrt{R_{WF}^{\,2} - \left(H_{core}/2\right)^2} \tag{13}$$

$$g_i = \sqrt{m^2 + j_i^{\,2}} \tag{14}$$

$$k_i = R_{WF} - g_i \tag{15}$$

The target path length, F_i, at each height increment, j_i, is no longer r_{max} but rather r_{max} less k_i.

$$F_i = r_{\max} - k_i \tag{16}$$

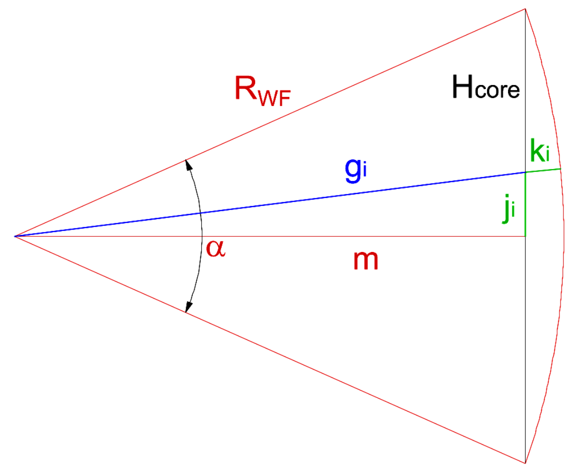

Figure 6: Geometry of side view to define a desired wave front curvature

5. DEFINING THE CONDUIT INSIDE THE PHASE PLUG

Thus far we have detailed how to create the shape of the inner core for the phase plug. We must also define the shape of the outer shell. The air space between the inner core and the outer shell creates a conduit through which sound waves may propagate.

In Figure 7, we see that the outer shell in the cross section shown is a constant distance *d/2* away from the inner core. However, this distance *d/2* is at some angle relative to the surface of the inner core. This angle will change depending on the angular cross section, φ, through the inner core. More specifically, the angle will change as $\theta_{TangentLine}$, (the angle of the line *t*) changes.

Typically this angle is different for each different angular cross section.

The offset distance can be more conveniently quantified by the distance perpendicular to the inner core. This distance is a function of the angle $\theta_{TangentLine}$ and is given by equation (18). To make the equations a bit simpler we will use beta, β, to represent $\theta_{TangentLine}$.

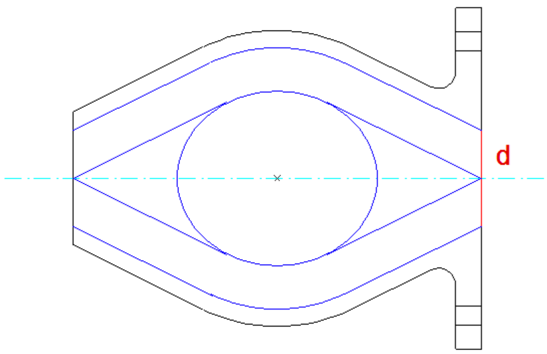

Figure 7: Top view of phase plug showing offset distance from the inner core to define the outer shell

$$\beta = \theta_{TangentLine} \tag{17 }$$

$$Offset = \frac{d}{2} * \cos(\beta) \tag{18 }$$

The ellipse for the outer shell in this cross section is also defined by offsetting the ellipse used for the inner core. The offset distance is simply added to the values for the semi-major and semi-minor axes, *a* and *b*, of the inner core ellipse.

$$a_{OuterShell} = a_{InnerCore} + Offset \tag{19 }$$

$$b_{OuterShell} = b_{InnerCore} + Offset \tag{20 }$$

There are two additional items that must be addressed to define the outer shell.

The first is that as the cross sections are taken at progressively greater angles through the inner core (Figure 8), an unmodified offset at the entry of the phase plug would result in a rectangular or square opening, not a circular opening as desired to mate with a compression driver or other generally circular loudspeaker drivers. The starting point of the tangent line, $t_{OuterShell}$, that defines the outer shell must be "tilted" a bit so that it will lie on the circular perimeter of the entry to the phase plug. Equation (21) is used to calculate the rotational angle where a given $t_{OuterShell}$ will intersect the perimeter of the circular entry

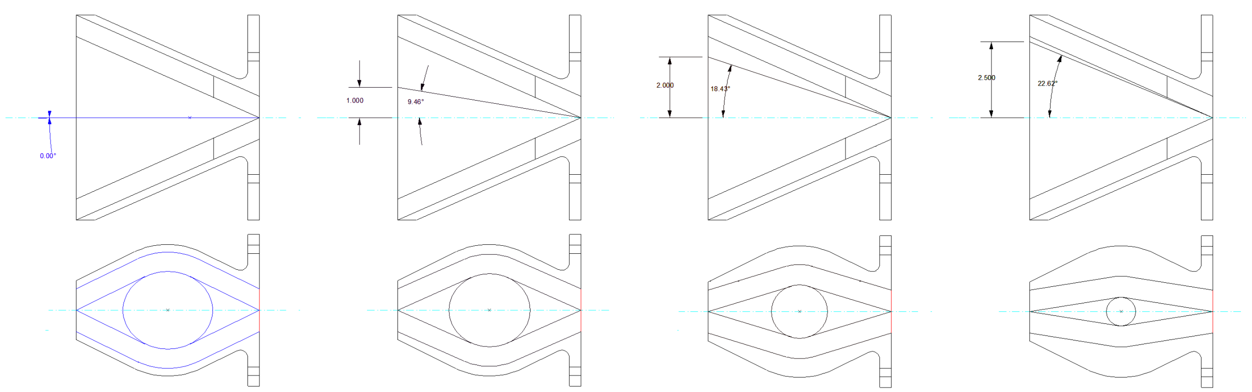

Figure 8: Different cross sections, φ, through the phase plug

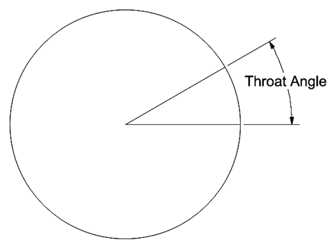

Figure 9: Throat Angle for the Outer Core

$$ThroatAngle = ThroatRatio * 90° \tag{21}$$

$$ThroatRatio = \varphi_n / \varphi_{\max} \tag{22}$$

Where and φ_n is the angular increment for a particular cross section.

In this manner, regardless of how many different angular cross sections, φ, are used to define the surface of the inner core the offset of each one is set proportionally at the proper place on the circular perimeter of the entry.

The point on an ellipse, where the tangent line intersects the ellipse, and defines the outer shell, is the second item to be addressed. The x and y coordinates of this point, in the plane of the angular cross section, are given by the equations (23) and (24)

$$x_{p\varphi} = p + Offset * \cos(\beta) \tag{23}$$

$$y_{p\varphi} = \frac{p}{e} - Offset * \sin(\beta) \tag{24}$$

Where $x_{p\varphi}$ is the lateral dimension and $y_{p\varphi}$ is the axial dimension within the plane of the angular cross section. The z coordinate would correspond to the height dimension.

6. PROTOTYPE MODELING & MEASUREMENTS

A prototype device was designed and fabricated using two identical phase plugs stacked above/below each other (Figure 10 and Figure 11). The exit of these phase plugs feed into a simple, straight wall horn designed for 10° vertical coverage, as might be used in a modular line array loudspeaker cabinet. Directivity measurements of the prototype were made at a distance of just over 6 m with an angular resolution of 2°. For the size of the device this should yield good data to about 8 kHz.

The on-axis frequency response is shown in Figure 12. This also shows a measurement of one of the compression drivers measured on a plane wave tube (PWT) for comparison. The overall level of the PWT measurement was adjusted to match that of the dual PDE prototype below about 2 kHz.

Vertical polar graphs, the vertical beamwidth, and a vertical directivity map are shown in Figure 13 through Figure 15. These show good results with a small amount of off-axis lobing. Refinements to the phase plug design and/or modifications to the horn mouth should improve this lobing.

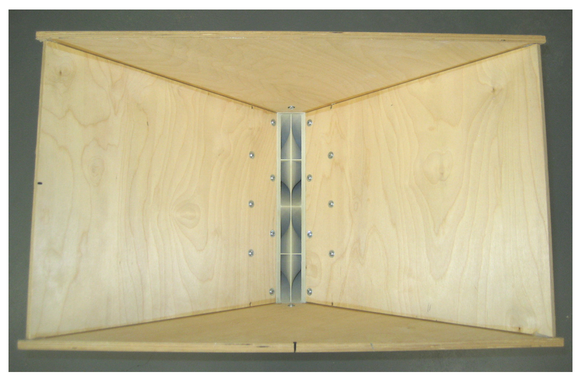

Figure 10: Front view of prototype and horn

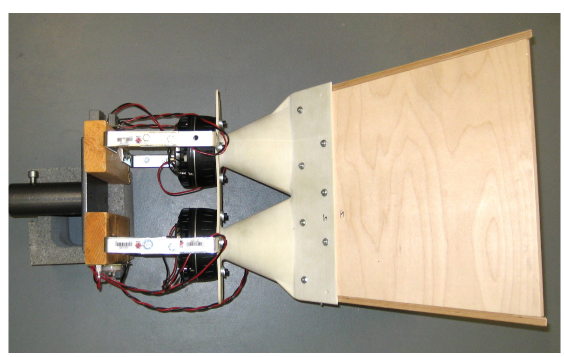

Figure 11: Side view of prototype and horn

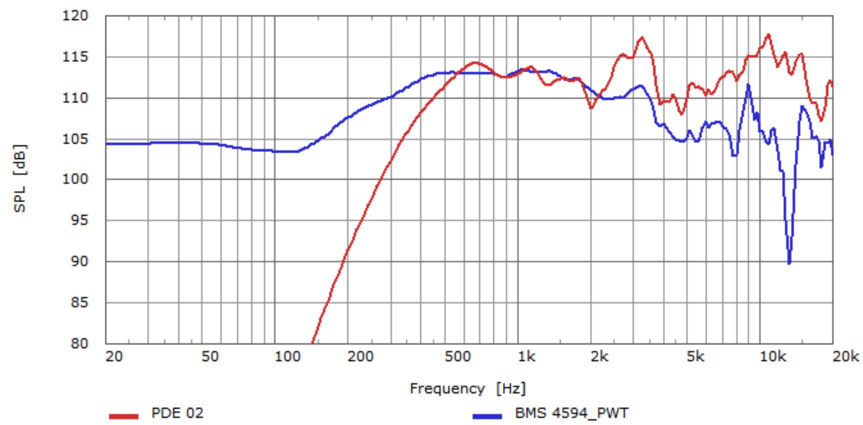

Figure 12: On-axis response of dual PDE prototype (red) and compression driver on plane wave tube (blue)

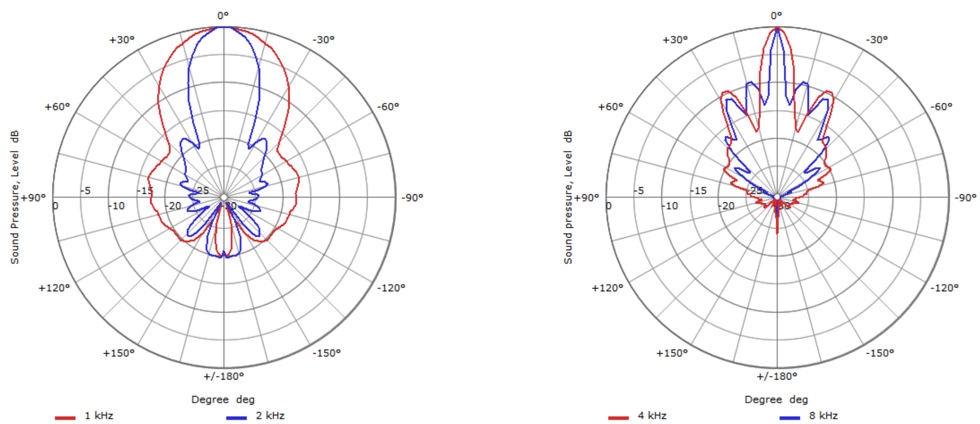

Figure 13: Vertical polar plots for dual PDE prototype at 1 kHz, 2 kHz, 4 kHz, and 8 kHz

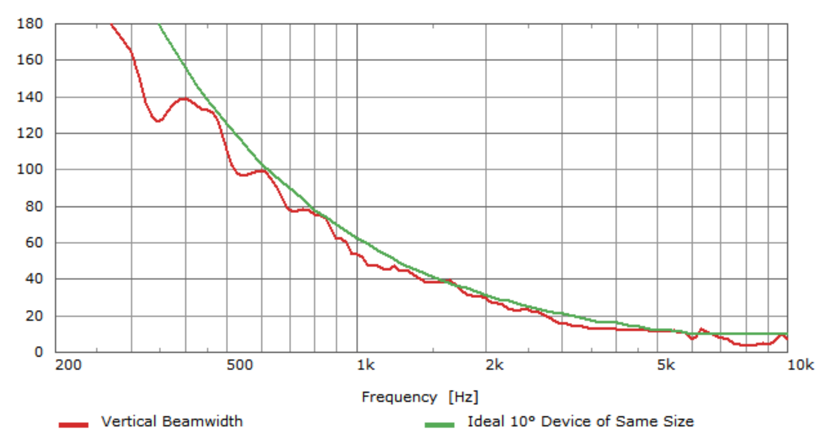

Figure 14: Vertical beamwidth of dual PDE prototype (red) and theoretical ideal 10° of the same size (green)

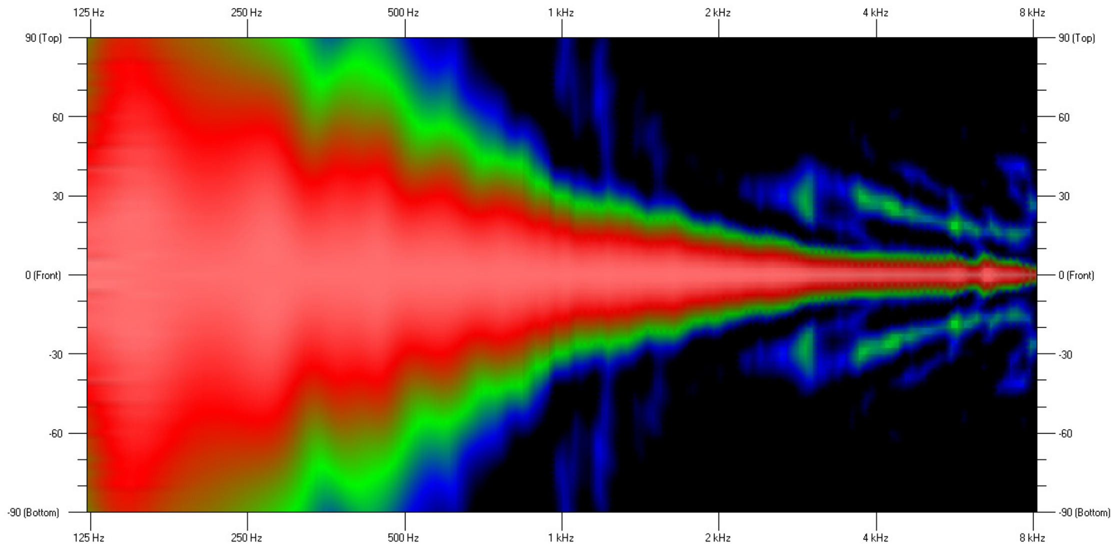

Figure 15: Vertical directivity map of dual PDE prototype device

7. ACKNOWLEDGEMENTS

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This technology is protected under U.S. Patent 8,887,862. International patents pending.

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